Monte Carlo simulations of a high-resolution X-ray CT system for industrial applications

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Abstract

An X-ray computed tomography (CT) model based on the GEANT4 Monte Carlo code was developed for simulation of a cone-beam CT system for industrial applications. The full simulation of the X-ray tube, object, and area detector was considered. The model was validated through comparison with experimental measurements of different test objects. There is good agreement between the simulated and measured projections. To validate the model we reduced the beam aperture of the X-ray tube, using a source–collimator, to decrease the scattered radiation from the CT system structure and from the walls of the X-ray shielding room. The degradation of the image contrast using larger beam apertures is also shown. Thereafter, the CT model was used to calculate the spatial distribution and the magnitude of the scattered radiation from different objects. It has been assessed that the scatter-to-primary ratio (SPR) is below 5% for small aluminum objects (approx. 5 cm path length), and in the case of large aluminum objects (approx. 20 cm path length) it can reach up to a factor of 3 in the region corresponding to the maximum path length. Therefore, the scatter from the object significantly affects quantitative accuracy. The model was also used to evaluate the degradation of the image contrast due to the detector box.

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1. Introduction

X-ray computed tomography (CT) provides quantitative, readily interpretable data and enables the inspection of structures that are not amenable to any other nondestructive evaluation technique. As a result, CT has become well established as an inspection, evaluation, and analysis tool in industry [1]. Many of the applications have been in the automotive industry, where mostly aluminum cast parts have to be inspected, but use in other industries is growing as equipment becomes increasingly available.

Nowadays there are different kinds of CT systems available on the market: fan-beam CT scanners using line detectors and cone-beam CT scanners using area detectors. Fan-beam scanners generate stacks of slices which have to be assembled to 3D volumetric data. The image quality obtained with such systems is high but the slice-by-slice scanning process requires long acquisition time and therefore is expensive [2]. Cone-beam CT scanners using area detectors allow simultaneous 3D data acquisition with shorter scanning time. This advantage of increased volume coverage is hampered by the growing contribution of Compton scatter radiation to the measured signal that leads to significant degradation of image quality. In fact, incident X-ray photons are scattered inside the object and,
if reaching the detector, cause an increase in the registered total intensity and lead to underestimation of the attenuation in each projection. The effect on CT images is a decreasing of the low-contrast detectability and the presence of cupping and streak artifacts between image regions of high attenuation [3–6].

Knowledge of scatter distribution is therefore essential to optimize the design and acquisition parameters of cone-beam CT systems for industrial applications. Moreover, the information about the scatter distribution can be used for corrective image reconstruction algorithms [7,8]. Monte Carlo (MC) based scatter correction for image reconstruction has already been performed in medical imaging [9].

The amount of scattered radiation depends strongly on the type of imaging system and detector geometry used as well as the object under study. Several authors have estimated the amount of scatter for medical scanners [4,5,10] while studies for industrial CT systems are rather limited [11–13]. Studies on medical imaging X-ray systems show that the scatter-to-primary ratio (SPR) increases with the object size and the field of view [4,10,11].

The scatter contribution in medical CT has been simulated by several authors using analytical models [14–16]. These models can be used with some success, but result often in approximate estimates of the true scatter distribution and therefore they are not suitable to simulate complex CT geometries. A more general and accurate way of estimating scatter contribution is to use MC simulation [3,10,11,17,18]. Although the investigation of scatter contribution using the MC method is time consuming compared to relatively simple mathematical modeling, the widespread availability of high performance parallel computing and Grid technology in addition to the popularity of variance reduction techniques spurred the use of MC calculations especially when modeling complex geometries [19].

The radiation scattered from the CT system structure and from the walls of the X-ray shielding room on the detector (environment scatter) is an additional source of noise and therefore artifact that affects the quality of the CT image. The amount of this radiation depends on the characteristics of the X-ray shielding room (dimensions and materials of the walls), the geometry, materials and working energy of the CT system, the aperture of the X-ray beam, and the materials and geometry of the investigated object. The environment scatter adds a uniform noise to the image that decreases the image contrast and therefore limits the low-contrast detectability [3]. The environment scatter is less critical than the scatter from the object because it does not change significantly with the projections. It can be assimilated to a constant signal and subtracted to the data. Moreover, it is possible to reduce the environment scatter placing the CT system in a bigger shielding room, choosing absorbing materials for the walls of the X-ray room and optimizing the design of the structure housing the detector.

In this work, a MC model for an X-ray CT system with an area detector configuration for industrial applications based on the GEANT4 code was developed. The simulation was designed to describe the image formation in CT starting from the generation of the X-ray photons up to their absorption in the sensitive detector. Detailed simulation of the X-ray tube, filters, test objects, and detector were considered. The model was validated by comparison with experimental measurements of test objects on a cone-beam CT system. The CT model was used to calculate the magnitude and spatial distribution of the scattered radiation from different objects. Since in this work our aim was the study of the scattered radiation from the object, we adopted all the necessary measures to reduce the environment scatter. The first was the use of a source–collimator that reduces the aperture of the X-ray beam to the region of interest, the second was the use of a lead shielding box around the X-ray tube to reduce the leakage radiation of the tube. In this paper we show the profile of a test object acquired using several kinds of source–collimator to show how the environment scatter, if not reduced, contributes to the degradation of the contrast on the image. Moreover, we evaluated, thanks to the MC simulation, the environment scatter produced by the only detector box.

2. Experimental setup and simulation procedure

2.1. CT system

The validity of the Geant4 based MC model was verified by comparing the simulated and measured distributions from various test objects on a high resolution cone-beam X-ray CT scanner for industrial applications.

The CT system (Fig. 1) consists of an X-ray source, an X-ray source–collimator, a detector composed of a scintillator, a mirror and a CCD camera, and a four axis manipulator. A filter of silver 1 mm thick (post-filter) is placed behind the scintillator screen.

![CT System Image](Image)

Fig. 1. Picture of the CT system. The X-ray tube and its housing, the object manipulator, the filter and the detector box are clearly visible. The scintillator screen is placed behind the filter; the CCD camera is inside the detector box.
placed in front of the scintillator at 1461 mm from the source. The scintillator, the mirror, and the CCD camera are placed inside a metallic box (detector box) of overall dimensions of $490 \times 510 \times 1000$ mm$^3$ (width $\times$ height $\times$ depth) with walls made of a 3 mm thick steel layer (inner layer), a 3 mm thick lead layer and a 5 mm thick steel layer (external layer). The source–detector distance is 1500 mm. A lead housing is placed around the X-ray tube to reduce the leakage of the X-ray tube housing. The dimensions of the X-ray shielding room are $3.6 \times 2.9 \times 3.7$ m$^3$ (width $\times$ height $\times$ depth). Two walls of the room are made of concrete and the other two of a sandwich of steel (3 mm), lead (25 mm), and steel (3 mm).

2.1.1. X-ray source

The X-ray unit employed is a 450 kV X-ray generator manufactured by Comet AG (Model MXR 451) with 2.3 mm iron and 1.0 mm copper inherent filtration. The angle of the target, made of tungsten, is 30°. The size of the focal spot is 2.5 mm. The emission cone of the X-ray source is 40°. A 1.0 mm tungsten (alloy: HPM1750) attenuator is used to reduce the primary flux of low-energy photons.

2.1.2. X-ray source–collimator

Three source–collimators were manufactured to investigate the influence of beam aperture on image contrast: (i) a 100 mm thick rectangular lead source–collimator with angular aperture $8.87° \times 6.09°$ (horizontal $\times$ vertical), (ii) a 180 mm thick brass source–collimator with angular aperture $5.61° \times 5.61°$ (big aperture), and (iii) a brass source–collimator with angular aperture $3.15° \times 3.15°$ (small aperture). The brass source–collimators can be inserted inside the lead source–collimator. Moreover the configuration without any source–collimator, which corresponds to a beam aperture of half angle 20°, is considered. In each studied configuration the test object was completely irradiated by the primary beam.

2.1.3. Detector

The X-ray converter is a Thallium-doped Cesium Iodine, CsI(Tl), scintillator with a thickness of 2 mm manufactured by Hamamatsu. The effective scintillator area is $480 \times 280$ mm$^2$. The 1 mm thick back plate is made of aluminum. The converted photons are projected on a 45° mirror and reflected on a CCD camera (Apogee Alta U32) of $2184 \times 1472$ pixels. The pixel size is $6.8 \times 6.8$ μm$^2$. A NIKON 28 mm lens is mounted on the CCD camera. The field of view is $524 \times 353$ mm$^2$.

2.2. Description of the GEANT4 based Monte Carlo CT simulation

The low-energy extension of the electromagnetic processes version 2.3 of the simulation toolkit GEANT4 [20–22] was used to model the interactions of photons and electrons with matter down to 250 eV. The processes activated in the physics list for electrons were: ionization, bremsstrahlung, and multiple scattering; for photons: Rayleigh scattering, Compton scattering, and photoelectric effect [23]. The cut value in range was set to 0.1 mm for photons and electrons.

The simulation is performed in two steps: the generation of the X-ray spectrum of the tube taking into account the anode angle, inherent and external filtration of the tube; and the generation of the projection of the object according to the acquisition setup (i.e. image of the energy deposited within the detector).

The simulations were run on a Pentium-IV-based personal computer with a 2.80 GHz microprocessor. The computing time to reach a good statistic strongly depends on the X-ray beam aperture; it goes from 1 day in case of small objects (approx. 5 cm path length, beam aperture of 6°) to several days in case of large objects (approx. 20 cm path length, beam aperture of 18°).

2.2.1. Generation of the X-ray spectrum

The generation of the spectrum involves the simulation of a monoenergetic pencil electron beam hitting the tungsten target at an angle of 30° with respect to the normal of the anode surface and the passage of the produced X-ray spectrum through inherent filtration (2.3 mm Fe and 1.0 mm Cu) and external filtration (1.0 mm W, alloy HPM1750). The radiation is retrieved within an angle of 20° with respect to the central axis of the beam. Fig. 2 shows the spectrum at 450 kV for the MXR-451 Comet X-ray tube simulated using the X-ray tube characteristics mentioned above. The spectrum was simulated with $2 \times 10^9$ primary electron histories. The validation of the simulated spectra has been assessed through comparison with experimental data [24].

2.2.2. Image of the deposited energy distribution

The X-ray photons are emitted from the focal spot of diameter 2.5 mm, with energy sampled randomly from the simulated spectrum, towards the object. Their direction is selected randomly from an isotropic distribution of angles

![Fig. 2. Simulated spectrum of the X-ray tube MRX-451. The electron energy was 450 keV, the inherent filtration was 2.3 mm Fe + 1.0 mm Cu and the external filtration was 1 mm W (alloy HPM1750).](image-url)
in a cone of selected aperture. When the X-ray photons reach the object they can undergo photoelectric effect, single Compton scattering, multiple Compton scattering, and Rayleigh scattering. The X-ray photons that leave the object in direction of the detector are filtered by the post-filter and the back plate of the scintillator. The photons interact with the scintillator by releasing energy to the material with production of electrons. When simulating the detector box, the photons that are not absorbed by the scintillator can interact with the detector box and backscatter into the scintillator. The energy and the position of production of the electrons within the scintillator together with the number of Compton and Rayleigh interactions of the parent photon within the object were retrieved. The CsI was simulated as a bulk material with density 4.52 g/cm³.

The scatter images were de-noised using the Richardson–Lucy fit. The procedure, which has been used by Colijn [12], utilizes a maximum likelihood algorithm to retrieve the original noise-free signal blurred by a Gaussian kernel. Smooth estimates of scatter projections can be obtained from simulation with a low number of photons. This allows reducing the time needed for MC simulations. In the present study, MC simulations were performed with \(10^9\) primary photon histories in case of small objects and \(2 \times 10^9\) in case of large objects. The projections were de-noised using 10 iterations of the Richardson–Lucy fit. The standard deviation of the Gaussian kernel was set to 30 detector pixels.

### 2.3. Evaluation and validation

**Measurements.** The X-ray high voltage was set to 450 kV and the current to 2 mA. The distance from the rotation axis to the detector plane was 226 mm. All the acquired images, \(I_{\text{acq}}(u,v)\), were normalized to remove some geometric effects and scanner non-uniformity, such as the spatial irregularity of the source radiation and the non-uniformity of the detector response, using the formula

\[
I_{\text{norm}}(u,v) = \frac{I_{\text{acq}}(u,v) - I_{\text{dark}}(u,v)}{I_{\text{air}}(u,v) - I_{\text{dark}}(u,v)}
\]  

where \(u\) and \(v\) are the detector coordinates, \(I_{\text{norm}}(u,v)\) is the normalized image, \(I_{\text{air}}(u,v)\) is the image in absence of the object and \(I_{\text{dark}}(u,v)\) is the image without X-rays. A \(4 \times 4\) binning was performed on the acquired images.

**Influence of the X-ray beam aperture.** To evaluate the influence of the X-ray beam aperture on the image contrast we manufactured an aluminum box of size \(50 \times 65 \times 50\) mm\(^3\) with two holes of size \(10 \times 10 \times 50\) and \(8 \times 10 \times 50\) mm\(^3\) along the axial direction with equal distance from the object center (Fig. 3a). Radiographies with both the brass source–collimators and the rectangular lead source–collimator were acquired. In addition, an acquisition without source–collimator was performed.

**Validation of the model.** Four test objects were used to validate the model: (i) the aluminum box, (ii) the aluminum box containing two copper rods, (iii) a cylinder made of aluminum of external diameter 66 mm and inner diameter 25 mm (small cylinder) (Fig. 3b), and (iv) a step wedge made of aluminum of overall dimensions \(100 \times 100 \times 20\) mm\(^3\) having five steps of thickness \([20–100]\) mm (Fig. 3c). To quantitative evaluate the MC simulation we calculated the root mean square (RMS) of the difference between the experimental and simulated projections given by the formula

\[
\text{RMS}_A = \sqrt{\left(\langle I_{\text{Exp}}(u,v) - I_{\text{Sim}}(u,v) \rangle_{(u,v)\in A} \right)^2} 
\]  

and we compared the result to the uncertainty (\(\sigma\)) defined by

\[
\sigma_A = \sqrt{\left(\langle I_{\text{Exp}}(u,v) - \langle I_{\text{Exp}}(u,v) \rangle_{(u,v)\in A} \rangle_{(u,v)\in A} \right)^2} + \sqrt{\left(\langle I_{\text{Sim}}(u,v) - \langle I_{\text{Sim}}(u,v) \rangle_{(u,v)\in A} \rangle_{(u,v)\in A} \right)^2} 
\]  

where \(u\) and \(v\) are the detector coordinates, \(I_{\text{Exp}}(u,v)\) is the experimental normalized image, \(I_{\text{Sim}}\) is the simulated image, and \(A\) is the region of interest (ROI).

**Scatter from large objects.** To evaluate the scatter from large objects we simulated a 200 mm high aluminum cylinder of external diameter 200 mm and inner diameter 50 mm. In this case the source–object distance was...
1000 mm and the pixel size was $1.92 \times 1.92 \, \text{mm}^2$. The post-filter was not simulated.

Degradation of the image contrast due to the detector box.

To evaluate the influence of the detector box on the image contrast, we carried out a simulation in the geometric configuration of Fig. 4. The detector box was modeled using the description given above. Also the rectangular source–collimator of lead was taken into account. The object was an aluminum step cylinder with inner diameter 20 mm, 8 external diameters (40, 60, 80, 100, 120, 160, 200, and 220 mm) and height of 160 mm. The full scintillator screen was irradiated by the primary beam. The source–object distance was set to 1000 mm. Thereafter, we performed a simulation with the same configuration but different composition of the walls of the detector box (3 mm thick lead layer and 5 mm thick steel layer) to evaluate the influence of the inner layer of the detector box on the image contrast.

Evaluation. The degradation of the image quality was evaluated by calculating the image contrast given by

$$C = \frac{I(u, v)_{(u, v) \in \text{flat}} - I(u, v)_{(u, v) \in \text{obj}}}{I(u, v)_{(u, v) \in \text{obj}}}$$

where $I(u, v)$ is the value of the gray level corresponding to the detector coordinates $(u, v)$, flat is a ROI in the image where the X-ray flux is not attenuated by the object and obj is a ROI in the image where the X-ray flux has been attenuated by the object.

The corruption of projection data by scattered photons was investigated by calculating the SPR for each order of scatter, $\text{SPR}_n(u, v)$, defined by

$$\text{SPR}_n(u, v) = \frac{S_n(u, v)}{P(u, v)}$$

and the total scatter-to-primary ratio, $\text{SPR}(u, v)$:

$$\text{SPR}(u, v) = \sum_n S_n(u, v) \frac{1}{P(u, v)} = \sum_n \text{SPR}_n(u, v)$$

where $S_n(u, v)$ is the value of the $n$-order scatter radiation corresponding to the detector coordinates $(u, v)$ and $P(u, v)$ is the value of primary radiation corresponding to the detector coordinates $(u, v)$. Moreover, the spatial distribution of the scatter was studied.

3. Results and discussion

In this section, the degradation of the image contrast due to the environment scatter and the validation of the Monte Carlo model are presented. Eventually the validated model is used to evaluate the scatter from the test objects and the degradation of the image contrast due to the detector box.

3.1. Degradation of the image contrast due to the environment scatter

As already mention in the introduction, the scattered radiation reaching the detector is composed of the scattered radiation from the object and the scattered radiation from the environment. The environment scatter depends on the size and wall materials of the X-ray shielding room, on the materials of the structure that houses the detector, on the X-ray spectrum of the source and, less strongly, on the object investigated. Since we were interested in studying the scatter produced by the object we adopted measures to reduce the environment scatter. We used a source collimator to reduce the aperture of the X-ray beam and a shielding box around the X-ray tube to reduce the leakage of the tube. The profile along a horizontal line of the test object, scanned using several source–collimators (Fig. 5a), shows how the environment scatter contributes to the degradation of the contrast on the image. The source–collimators used were: (i) the brass source–collimator of small aperture, (ii) the brass source–collimator of big aperture, and (iii) the rectangular lead source–collimator. Moreover an acquisition without any source–collimator was performed. The Fig. 5(b) shows that the value of the contrast decreases considerably with the aperture of the X-ray beam: it goes from 2.60 when the brass source–collimator of small aperture is used ($3.15^\circ \times 3.15^\circ$) to 1.60 when no source–collimator is used.
The small dimensions of the X-ray shielding room, the material of the walls (lined with steel 3 mm thick), the high energy of the X-ray beam, and the CT system structure are the cause of this considerable amount of environment scatter. The environment scatter can be reduced by placing the CT system in a bigger shielding room, optimizing the composition of the walls to reduce the scattering and optimizing the structure that houses the detector. When the system is not modifiable and already installed in a shielding room the environment scatter can be evaluated by acquiring a radiography of an object easy to simulate, similar to the one we want to analyze and performing the simulation with the same parameters of the experiment. The difference between the results represents the environment scatter.

In paragraph 3.4 we simulated the environment scatter due only to the detector box to study the influence of the detector box on the degradation of the contrast.
3.2. Validation of the simulation

Figs. 6–9 show the comparison of simulated and measured profiles, after normalization, for various test objects. The profiles were calculated from the ROI AA'. There is generally good agreement between the simulations and the experimental results. The values of the RMS of the difference between the simulated and experimental data calculated for the ROIs defined in Figs. 6–9 were compared to the combined uncertainties (Table 1). Fig. 6 shows the simulated (a) and measured (b) image of the aluminum box together with the comparison of measured and simulated profiles presented in log-linear scale to magnify the differences between simulated and measured results (c). Although the profile calculated from the simulated image is noisy it appears clearly that the simulated and measured profiles are in agreement. The values of the RMS calculated from the profile of the aluminum box in correspondence to the maximum path length of the object and the void hole (Table 1, ROIs a–c) are smaller than the combined uncertainties. Fig. 7 shows the normalized simulated (a) and measured (b) image of the Al box containing the Cu rods, and the comparison of simulated and measured profiles. The comparison between the RMS and the uncertainties corresponding to the region of maximum path length of the object and to the region with Cu rods (Table 1, ROIs d–f) shows that the simulated and measured data are in agreement. Fig. 8 shows the simulated (a) and measured (b) image of the Al cylinder, and the comparison of simulated and measured profiles of the central ROI. The values of the RMS selected from the profile, corresponding to the maximum path length and to the center of the object, are smaller than the uncertainties.

<table>
<thead>
<tr>
<th>ROI</th>
<th>$x_1$ (mm)</th>
<th>$x_2$ (mm)</th>
<th>RMS ($10^{-2}$)</th>
<th>Uncertainty ($10^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>–25</td>
<td>–10</td>
<td>0.74</td>
<td>1.0</td>
</tr>
<tr>
<td>b</td>
<td>10</td>
<td>25</td>
<td>0.59</td>
<td>0.6</td>
</tr>
<tr>
<td>c</td>
<td>–3</td>
<td>3</td>
<td>1.71</td>
<td>3</td>
</tr>
<tr>
<td>d</td>
<td>–25</td>
<td>–10</td>
<td>0.71</td>
<td>1.0</td>
</tr>
<tr>
<td>e</td>
<td>9</td>
<td>24</td>
<td>0.64</td>
<td>1.0</td>
</tr>
<tr>
<td>f</td>
<td>–3</td>
<td>2</td>
<td>0.29</td>
<td>2.0</td>
</tr>
<tr>
<td>g</td>
<td>–3</td>
<td>3</td>
<td>0.65</td>
<td>0.8</td>
</tr>
<tr>
<td>h</td>
<td>–22</td>
<td>–16</td>
<td>0.48</td>
<td>2.0</td>
</tr>
<tr>
<td>i</td>
<td>16</td>
<td>22</td>
<td>0.68</td>
<td>2.0</td>
</tr>
<tr>
<td>j</td>
<td>–52</td>
<td>–37</td>
<td>0.80</td>
<td>1.0</td>
</tr>
<tr>
<td>k</td>
<td>–30</td>
<td>–15</td>
<td>0.74</td>
<td>0.9</td>
</tr>
<tr>
<td>l</td>
<td>–7</td>
<td>8</td>
<td>0.47</td>
<td>0.6</td>
</tr>
<tr>
<td>m</td>
<td>17</td>
<td>34</td>
<td>0.46</td>
<td>0.6</td>
</tr>
<tr>
<td>n</td>
<td>39</td>
<td>48</td>
<td>0.57</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 1
Root mean square of difference between experimental and measured projections (RMS) and uncertainty for the selected ROIs.

Fig. 8. Comparison between simulated and experimental radiographies for the Al cylinder. (a) Simulated image. (b) Measured image. (c) Profile of the Al cylinder, corresponding to the ROI AA'.

Fig. 9. Comparison between simulated and experimental radiographies for the Al step wedge. (a) Simulated image. (b) Measured image. (c) Profile of the Al step wedge corresponding to the ROI AA'.
Fig. 9 shows the normalized simulated (a) and measured (b) image of the Al step wedge together with the comparison of simulated and measured profiles of the central ROI. Although the simulated profile appears to be slightly higher than the measured profile in correspondence to the step 100 mm thick, the Table 1, ROI n, shows that the intensities are in agreement within the uncertainty limits. This is due to the fact that the flux is highly attenuated and the signal-to-noise ratio (SNR) is low. For an accurate convergence of the result, it is necessary to simulate a larger number of photon histories.

3.3. Magnitude and spatial distribution of scatter radiation from the object

Evaluation of scatter from small objects. The profile of first-order scattering, second-order scattering, and higher-orders scattering is shown in Fig. 10a for the aluminum box. The profile of the first-order scattering follows the shape of the test object. The profiles of the second and higher-order scattering are approximately uniform. Fig. 10b–10e show the SPR$_{1}$, SPR$_{2}$, and SPR$_{>2}$ for the aluminum box, respectively, without and with copper rods, the small cylinder, and the step wedge. As expected, the major contribution to the scattering is given by the first-order scattering. The SPR$_{1}$, SPR$_{2}$ and SPR$_{>2}$ are peaked in the region corresponding to the maximum path length because of the smaller probability of transmitted primary photons. In the area corresponding to the maximum path length the value of the SPR$_{1}$ for aluminum box is below 4%, SPR$_{2}$ and SPR$_{>2}$ are both below 1% (Fig. 10b). The SPR$_{1}$ for the aluminum box containing copper rods is 12% in the area corresponding to the copper rod and below 4% in the area corresponding to the aluminum, the

![Fig. 10. (a) Simulated profiles of the first, second, and higher than second order scattered photons of the Al box. Scatter-to-primary ratio (SPR) for the projection of (b) the Al box, (c) the Al box containing copper rods, (d) the Al cylinder, and (e) the Al step wedge.]
SPR$_2$ is 3% in correspondence to the copper rod and below 1% to the aluminum, while SPR$_{>2}$ is below 1% in correspondence to the copper rod and to the aluminum (Fig. 10c). The SPR$_1$ for the small cylinder is 6.5% in the area where the path lengths are maximum and 4.5% in the central area, the SPR$_2$ is, respectively, 1.2% and 1%, and SPR$_{>2}$ is below 1% (Fig. 10d). The SPR$_1$ for the step wedge is 3.5% in the area corresponding to the maximum path length, the SPR$_2$ and SPR$_{>2}$ are below 1% (Fig. 10e).

**Evaluation of scatter from large objects.** CT of large aluminum objects, such as the aluminum cylinder of maximum path length 150 mm, is a challenge because of the low SNR. The probability for a photon to be transmitted without interaction through the cylinder is 1.1% (the equivalent energy of the X-ray spectrum is 250 keV). Moreover the scattered radiation, having low energy, more likely interacts with the scintillator than the primary photons. The images of the energy deposited by total radiation, primary radiation (i.e. photons that do not interact within the object before being detected), first-order scattered radiation, second-order scattered radiation, and higher-order scattered radiation and the corresponding profiles are displayed in Fig. 11. Due to the large object size, the multiple scattering plays a major role and in the region corresponding to the maximum path length its contribution to the image is higher than the signal. SPR$_1$ is 30% in the center and 80% in the region corresponding to the maximum path length, SPR$_2$ is 20% in the center and 55% in the region corresponding to the maximum path length.

<table>
<thead>
<tr>
<th>Orders of scattering included</th>
<th>$C_{\text{center}}$</th>
<th>$C_{\text{Max. path length}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>93 ± 8</td>
<td>250 ± 30</td>
</tr>
<tr>
<td>1st</td>
<td>72 ± 5</td>
<td>140 ± 10</td>
</tr>
<tr>
<td>1st + 2nd</td>
<td>63 ± 4</td>
<td>113 ± 8</td>
</tr>
<tr>
<td>All</td>
<td>48 ± 2</td>
<td>73 ± 5</td>
</tr>
</tbody>
</table>

Table 2
Contrast calculated from the profiles shown in Fig. 11f
length and SPR >2 is 50% in the center and 140% in the region corresponding to the maximum path length. The degradation of the contrast due to the n-order of scattering in the center of the object and in the region corresponding to the maximum path length calculated from the profile in Fig. 11f is shown in Table 2.

3.4. Degradation of the image contrast due to the detector box

The image of the step cylinder obtained when the detector box is simulated is displayed in Fig. 12(a). The Figs. 12(b) and (c) show the comparison of the profile extracted from the image in Fig. 12(a) with the profiles obtained when the detector box is not simulated. The profile obtained in the same condition of Fig. 4 but considering the walls of the box composed of two layers (3 mm lead, 5 mm steel) is also shown. The value of the attenuated intensity in the region corresponding to the maximum path length is 0.02% with the detector box, 0.01% without the detector box and 0.01% with the detector box without the inner layer. We conclude that the inner layer of steel is the major responsible of the production of the scatter. Table 3 shows the contrast calculated from the central vertical profile (Fig. 12b) for each step. The degradation of the contrast due to the detector box with walls composed of three layers is 68% in correspondence to the ring of diameter 220 mm. Due to the high attenuation, it was not possible to distinguish between the ring of diameter 200 and 220 mm.

4. Conclusions

A Monte Carlo model of a cone beam X-ray CT system for industrial applications has been developed and experimentally validated using CT scans of several test objects. The simulated projections were in good agreement with the measured data. A source–collimator was used to reduce the environment scatter. The degradation of the contrast due to the environment scatter was found to be 61.5% when no source–collimator is used. The model was used to calculate the images of the first-order scattering, second-order scattering, and higher-order scattering for different objects. The total contribution of the scattered radiation to the signal in case of small aluminum objects is lower than 5%. In case of objects of large dimensions, such as the aluminum cylinder of diameter 200 mm, the total contribution of the scattered radiation to the signal from

Table 3
Contrast calculated from the profiles in Fig. 12b

<table>
<thead>
<tr>
<th>Diameter ring (mm)</th>
<th>C_{Detector box}</th>
<th>C_{Detector box without inner layer}</th>
<th>C_{Without detector box}</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.89±0.04</td>
<td>0.90±0.05</td>
<td>0.92±0.04</td>
</tr>
<tr>
<td>60</td>
<td>2.46±0.09</td>
<td>2.5±0.1</td>
<td>2.54±0.09</td>
</tr>
<tr>
<td>80</td>
<td>5.0±0.2</td>
<td>5.3±0.2</td>
<td>5.3±0.2</td>
</tr>
<tr>
<td>100</td>
<td>9.3±0.4</td>
<td>9.9±0.4</td>
<td>9.9±0.4</td>
</tr>
<tr>
<td>120</td>
<td>15.5±0.9</td>
<td>17.2±0.9</td>
<td>17.6±0.9</td>
</tr>
<tr>
<td>160</td>
<td>34±2</td>
<td>41±3</td>
<td>42±2</td>
</tr>
<tr>
<td>220</td>
<td>53±3</td>
<td>85±6</td>
<td>78±5</td>
</tr>
</tbody>
</table>
the object itself is 280% and leads to degradation of the image contrast of 30%. Therefore, the evaluation of the multiple scattering for large objects is of primary importance for the optimization of the CT system and for the scattering correction. Moreover, the developed CT model enabled the evaluation of the degradation of the contrast due to the photons backscattered from the detector box onto the detector. The inner layer of 3 mm steel was found to be the major source of scatter from the detector box. The developed CT model is therefore a versatile tool to evaluate the contribution of scattered radiation to the image and to optimize the CT system design.

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References